SPACE-TIME BLOCK CODES (STBC)
FOR 4 TRANSMIT ANTENNAS.

Aresh Daruwalla
School of Electrical Engineering and Telecommunications, University of New South Wales, Australia
(a.daruwalla@zmail.unsw.edu.au)

Allen Paul
School of Electrical Engineering and Telecommunications, University of New South Wales, Australia
(allen@unsw.edu.au)

ABSTRACT

In modern wireless communications, numerous diversity techniques are used to improve the performance of signal transmission over multiple channels. This paper focuses primarily on the evaluation of Quasi Orthogonal & Orthogonal Space-Time Block Codes (STBC) for a 4x2 system model. The paper proceeds by checking which of these two codes - Orthogonal STBC or Quasi-orthogonal STBC, is better by evaluating relationship between the Symbol Error Rate (SER) and Signal to Noise Ratio (SNR). Furthermore we shall evaluate the performance of each with respect to their diversity.

Keywords: Space-time Block Codes (STBC), Symbol Error Rate (SER).

1 INTRODUCTION

Space-Time Block Codes (STBC) is an efficient technique of sending data over a wireless channel. A number of antennas are used at the transmitter. In STBC multiple copies of the same data are sent over a number of different antennas according to the given codewords. The medium of transmission being wireless, the signals sent may suffer from scattering and also due to reflection of the signal from different objects. These reflected copies do not manage to reach the receiver antenna at the same time. Furthermore due to channel noise some of these copies may get corrupted during the way. The solution to this is redundancy (ie. diversity). Multiple copies got at the receiver end provide redundancy which may lead us to eliminate the effect of noise providing us with optimum solution at the receiver.

.

To overcome the effects of fading and reflection an important scheme used is diversity. Diversity techniques may exploit the multipath propagation, resulting in a diversity gain [3]. Diversity is of two prominent types. One is transmitter diversity and receiver diversity. Diversity increases the probability of reliability of the signal at the output. The scheme utilized in our project is Time and Space Diversity. In time diversity multiple versions of the same signal are sent over different time slots and Space diversity refers to the copies being sent over different antennae. Transmitter diversity is achieved by using several antennas to transmit the signal. Antennae at the transmitter should be placed far apart from each other so that the signals do not correlate when they are received by the receiver. To achieve transmitter diversity we need to code our data to be sent over the channel using a coding scheme in which the codes are orthogonal to each other so that the signals may not interfere with each. Transmitter diversity is difficult to achieve.

In order to achieve receiver diversity signals from two separate antennas are used to reduce the impact of spatial variations in signal strength thus increasing the average data rate which is available.

2 SYSTEM MODEL

Our system model consists of four antennas at the transmitter and two antennas at the receiver which is given in Fig [1].

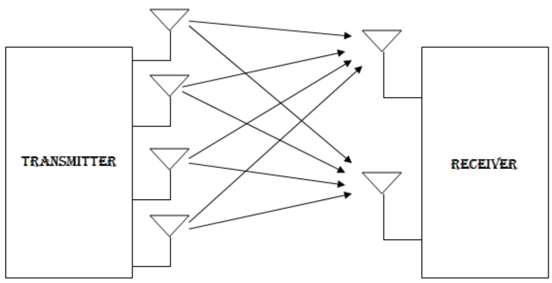

Figure 1: A 4 x 2 Communication System

Consider "M" symbols - x_1, x_2 …. x_M to be transmitted over a channel with the overall channel impulse response "H". The noise in the channel is denoted by "n". The system of equations is given by (1) and (2):

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

$$\begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} h_{1,1} & \cdots & h_{1,M} \\ \vdots & \ddots & \vdots \\ h_{N,1} & \cdots & h_{N,M} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_M \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_N \end{bmatrix} \tag{2}$$

Where:

- M = # of antennas at the Transmitter.
- N = # of antennas at the Receiver.
- $h_{N,M}$ = Path gain from transmit antenna N to receiver antenna M.

Our channel is a Gaussian channel with constant Rayleigh fading. Hence, each noise samples is independent with zero-mean. Noise is also a Gaussian random variable. Hence, each output "y" is also Gaussian. The path gain is designed to be samples of independent complex Gaussian Random variables. The variables have their real and imaginary parts to have 0.5variance.

In our following scenarios we shall denote the input symbols by "s". y_1, y_2, y_3, y_4 are the respective received signal vectors for the time instants t_1, t_2, t_3, t_4 for the first antenna at the receiver while y_5, y_6, y_7, y_8 are the respective received signals at the second antenna at the receiver.

3 ASSUMPTIONS

In our project we have made 3 major assumptions.

1. We have assumed an independent and ideally distributed (i.i.d) Rayleigh fading channel.
2. The noise we are considering in our transmission channel is purely Gaussian Noise.
3. We shall implement Maximum Likelihood (ML) decoding which means that each symbol at the input is thought to have the same probability of occurrences.

4 SCENARIOS

4.1 Scenario 1: For Orthogonal Space-Time Block Codes (STBC)

The first thing we need to make clear is why we call our codes Orthogonal Space-time Block Codes and why not just Space-time Block Codes. Consider a matrix "H". Orthogonal STBC have the property that the matrix H is orthogonal only if its transpose is equal to its inverse as shown in (3) and (4).

$$HH^T = H^T H = I \tag{3}$$

$$H^T = H^{-1} \tag{4}$$

We need to prove a property of Orthogonal STBC which states that Orthogonal Space-time Block codes always have BER less than one [5]. In Scenario 1, we have considered the transmission of QPSK symbols over a 4x2 channel with independent and ideally distributed (i.i.d.) Rayleigh fading given by (4).

Usually STBCs for real signal constellations are constructed from generalized orthogonal designs. For lower rate designs, we however replace a column with zeros. These are in short also known as Unitary Designs.

$$\mathbf{C} = \begin{pmatrix} s_1 & -s_2^* & -s_3^* & 0 \\ s_2 & s_1^* & 0 & -s_3^* \\ s_3 & 0 & s_1^* & s_2^* \\ 0 & s_3 & -s_2 & s_1 \end{pmatrix} \tag{5}$$

In the above codeword, the main diagonal is kept zero i.e. the various symbols are sent from three different transmitter antennae while the fourth antenna sends nothing. Thus in four time slots the system sends only three symbols. Thus unlike Alamouti STBC, this is not a full rate code. This is a rate 3/4 code.

4.2 Scenario 2: For Quasi-orthogonal Space-Time Block Codes (STBC)

Once again consider the transmission of QPSK symbols over a 4x2 channel with independent and ideally distributed (i.i.d.) Rayleigh fading. In Scenario 2, we have considered Quasi-orthogonal Space-time Block Codes for four transmit antennas given in (6).

$$C = \begin{pmatrix} A & -B^* \\ B & A^* \end{pmatrix} = \begin{pmatrix} s_1 & -s_2^* & -s_3^* & s_4 \\ s_2 & s_1^* & -s_4^* & -s_3 \\ s_3 & -s_4^* & s_1^* & -s_2 \\ s_4 & s_3^* & s_2^* & s_1 \end{pmatrix} \quad (6)$$

The above codeword has a rank of 2. From the above, we can see that different symbols are sent over different antennae and the conjugates of the symbols are sent at the 2nd and the 3rd time slots. This codeword can achieve diversity of 4. The rate of the code is one.

5 DIVERSITY GAIN

The relation between the error rate and the diversity order is given by the relation below

$$\overline{P}_s = c\overline{\gamma}^{-M} \quad (7)$$

Where:
c = Constant that depends on the modulation and coding.
γ = Average received SNR [6].

The Diversity Gain is the M-fold increase in the SNR performance due to the diversity order of the various schemes. Here, we observe that the diversity Gain for Scenario 1 is three. However, the diversity gain for Scenario 2 is four.

6 SYMBOL RATE

The Symbol Rate of any block code is given by the formula:

$$Symbol\ Rate = \frac{No.of\ Symbols\ Transmitted}{No.of\ Time\ Slots} \quad (8)$$

6.1 For Scenario 1:

In Scenario 1 for Orthogonal SBTC we are transmitting three symbols in four time slots. Hence the code rate for Orthogonal STBC is given by (10):

Code Rate for Orthogonal STBC= ¾ (9)

6.2 For Scenario 2:

In Scenario 2 for Quasi-Orthogonal SBTC we are transmitting four symbols in four time slots. The Code Rate for Quasi-orthogonal STBC is given by (10).

Code Rate for Quasi−Orthogonal STBC=4/4 (10)

7 PERFORMANCE

The performance of the block codes was studied and it is shown below.

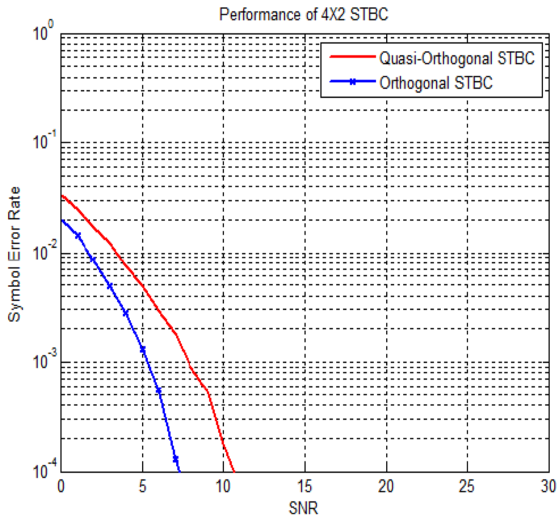

Figure 2: MATLAB simulation output between SER and Signal to Noise Ratio.

Hence, we infer that the performance of Orthogonal STBC is significantly better than quasi-orthogonal STBC. The Symbol error rate for orthogonal STBC drops to zero in under 5 dB SNR whereas it takes 10 dB longer for Quasi-Orthogonal STBC to do so.

	Orthogonal STBC	Quasi-orthogonal STBC
Diversity Gain	3	4
Symbol Rate	3/4	1

Table 1: Performance comparison.

Hence from the simulation and the graphs, we can infer that:

1. The average symbol error probability decreases as the diversity order (i.e. the number of the receiver antenna) in the system increases.

2. The performance of Orthogonal STBC is better than Quasi-Orthogonal STBC. But,

3. The symbol rate of the Orthogonal STBC is lower than the Quasi-Orthogonal STBC and the diversity gain for orthogonal STBC is also lower than Quasi-Orthogonal STBC.

8 CONCLUSION

After carefully analyzing both Orthogonal Space-Time Block Codes and Quasi-orthogonal Space-time Block Codes we can conclude that the Average Error probability Performance of Orthogonal STBC is better even though it has lower diversity gain and Symbol Rates than the Quasi-Orthogonal STBC both the codes is the same.

9 REFERENCES

[1] O. Tirkkonen and A. Hottinen, "Square-matrix Embeddable Space-time Block Codes for Complex Signal Constellations," IEEE Transactions on Information Theory, Vol. 48, pp. 1122–1126, Feb. 2002.

[2] H. Jafarkhani, "A quasi-orthogonal space-time block code," IEEE Transactions on Communications., Vol. 49, pp. 1–4, Jan. 2001.

[3] Weifeng Su and Xiang-Gen Xia, "Signal Constellations for Quasi-Orthogonal Space–Time Block Codes With Full Diversity." IEEE Transactions on Information Theory, Vol. 50, No. 10, October 2004.

[4] Marsch, P. Rave, W. Fettweis, G. "Quasi-Orthogonal STBC Using Stretched Constellations for Low Detection Complexity." Wireless Communications and Networking Conference, 2007.WCNC 2007. IEEE. Publication Date: 11-15 March 2007. Page(s): 757-761.

[5] Wireless Communications, Andrea Goldsmith.