SEMANTIC MODELLING OF CONTEXT AWARE SYSTEMS IN A LOGICAL FRAMEWORK

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ABSTRACT

The paper presents a logical framework for the modelling of context aware systems. The framework consists of three first order languages that together make it possible to represent all aspects of such systems and which thus provide a transparent modelling framework. The framework is constructed for the use in semantic modelling of context aware systems and models can for most parts easily be implemented in OWL/SWIRL. In addition to presenting the three languages an account is given on how to model a system, i.e. how the different elements of a context aware system are to be represented by the symbolic elements of the languages.

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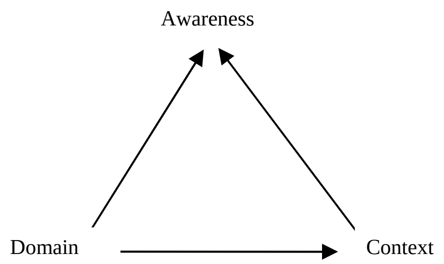
1 INTRODUCTION

Context Awareness is naturally considered in relation to a Domain and the relations between the conceptual content of the three terms are pictured by the semiotic triangle



that should be interpreted cognitively to say that awareness of the structure of a domain consists in its contextualisation. The being that possess such a contextualisation is then capable of "reason-able" actions with respect to predefined aims. The semiotic triangle pictures three distinct semantic levels which a modelling framework for context aware systems must take into account.

In the following I will present a generic framework for the modelling of such systems. It consists of three separate but interconnected first order languages: object language, metalanguage and property language. The object language describes the objects of the domain. The property language describes the properties of the objects, i.e. the predicates in the object language are names in the property language; while the object language provides the language to describe the empirical fact about the objects of a domain, the property language provides the language for the formulation of the formal part of a scientific theory. The metalanguage describes the semantic relations between the domain and the object language. Together, the three languages make it possible to represent the semantic levels pictured by the semiotic triangle and to state rules determining actions triggered by awareness.

With respect to an *intensionally* interpreted object language, the constructions of the property language and the metalanguage are canonical. An intensional interpretation conceives that the structure of the domain is mapped into the language [1]. The opposite conception is the extensional one which conceives that the structure of the language is mapped into the domain [2]. The direction of the mapping has consequences for the modelling of the domain as well as on the concept of truth that is determined by the verification of atomic propositions.

2 MODELLING FRAMEWORK [3,4]

Individuals possess properties and relations and the attribution of a property to an individual or a relation to two individuals constitutes an atomic fact about the individual or individuals. It is expressed by an atomic sentence, i.e. atomic sentences are alleged atomic propositions or statements about observed atomic facts.

The measurements of atomic facts about a single individual all involve the use of a standard of measure. The result of a measurement follows from a comparison between a representation of the standard and the individual. It determines a value from the standard (a predicate of the first kind).

Measurements are based on operational definitions, i.e. definitions that specify the applied standard of measure, the laws/rules on which the measurements are based and the instruction of the actions to be performed to make a measurement. The operational definitions provide intensional interpretations of the predicates expressing results of measurements. The measurement of the colour of a system is an example. The measuring device is then a colour chart where each of the colours is named and the rule of application is to compare the colour of the system with the colours on the colour chart and pick out the one identical to the colour of the system. The name of the colour picked denotes the result of the measurement.

Each operational definition defines a kind of measurements that is symbolised by an observable simulating the act of measurement; the observable is a map from the domain to the standard of measure that maps an individual to the value representing a property possessed by the individual [5]. The set of possible values of an observable represent mutually exclusive properties of the individuals of the domain; no two properties corresponding to different values of the same observable can be possessed by any individual. An individual cannot at the same time weight 1 kg and 2 kg. Weight is therefore an observable. Other examples of observables are position in space, temperature, number of individuals and colour.

The observation of relations is also simulated by maps that will be called observables; in fact, each kind of relation is associated with an operational definition. Particular relations and individuals being elements of the domain have the same ontological status, while properties and kinds of relations share epistemological status. I will indicate the reference to kind of relations and relations by the superscript $^{(2)}$ when necessary.

2.1 Object Language

Let $L_D(N_1 \cup N^{(2)} \cup V, P_1 \cup P^{(2)} \cup P_2)$ stand for the object language for a domain D. It consists of a vocabulary, the names of individuals N_1 and relations $N^{(2)}$, variables V, 1-ary predicates of the first kind P_1, 2-ary predicates $P^{(2)}$ referring to kinds of relations, 1-ary predicates of the second kind P_2 and logical connectives, and also sentences and formulae formed as syntactically acceptable combinations of the elements of the vocabulary. The distinction between 1-ary predicates of the first and second kind is semantic and made possible by the intensional interpretation. The predicates of the first kind are primary terms those of the second kind are introduced by terminological definitions. The sentences describe the objects of D, individuals and relations. D is modelled as a directed graph [6,7][1]. A node, endowed with an internal structure, represents an individual while an arrow (edge) $\mapsto$ with the source and target nodes stands for a relation r between the corresponding individuals.

The naming of the individuals and relations is symbolised by a map ν [2],

$$\begin{aligned} \nu : D \to N_1 \cup N^{(2)}; \; d &\mapsto \nu(d) = n \\ r &\mapsto \nu(r) = (n_s, n_t) \end{aligned} \tag{1}$$

that to an individual d in the domain D associates the name n by $\nu(d) = n$ or to a relation r the name (n_s, n_t) by $\nu(r) = (n_s, n_t)$ where s and t refer to the source and target of the arrow depicting the relation. ν is an isomorphism; by convention, there is a unique name for every individual or relation and each name refers to a unique individual or relation.

An observable δ simulate the determination of an atomic fact about an individual $d \in D$ or relation $r \in D$ by the associated kind of measurements,

$$\begin{aligned} &\delta : D \to P_1; \; d \mapsto \delta(d) = p \\ &\delta^{(2)} : D \to P^{(2)}; \; r \mapsto \delta^{(2)}(r) = p^{(2)} \end{aligned} \tag{2}$$

Moreover, for each observable δ (or $\delta^{(2)}$) there exists a unique map π (or $\pi^{(2)}$) defined by the condition of commutativity of the diagrams

$$\begin{array}{ccc} N & \xrightarrow{\pi} & P \\ \nu\uparrow & \nearrow \delta & \\ D & & \end{array} \tag{3}$$

i.e. $\delta(d) = \nu(\pi(d)) \quad \forall d \in D$

where N, P and δ stands for either N_1, P_1, δ and π or $N^{(2)}$, $P^{(2)}$, $\delta^{(2)}$ and $\pi^{(2)}$.

The diagrams relate the simulation of observations determining atomic facts assigning a property to an object or a relation to a pair of objects and the formulation of atomic sentences expressing these facts. The commutativity of the diagrams thus expresses truth conditions. In fact, if $n = \nu(d)$ and $p = \pi(d)$ then "pn is true", i.e. ""n is p" is true".

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[1] The domain D is throughout identified with its symbolic model.

[2] Note that the arrows $D \to N_1 \cup N^{(2)}$ and $d \mapsto n$ in the equations (1) stands for kinds of relations and relations in the metalanguage.

2.2 Property language

Predicates of the first kind refer to properties of systems. A property is something in terms of which a system manifests itself and is observed, and by means of which it is characterised and identified. To an observer a system appears as a collection of properties. The properties of a system are thus in a natural way mentally separated from the system. The separation is made possible by the fact that the ‘same’ property is possessed by more than one system. The separation is expressed by the commutativity of the following diagrams, each of which can be considered as a collection of semiotic triangles

$$\begin{array}{ccc} & & P_1 \\ & \delta\nearrow & \uparrow\rho \\ D & \xrightarrow[\varepsilon]{} & E \end{array} \qquad (4)$$

i.e. $\delta(d) = \rho(\varepsilon(d)), \quad \forall d \in D$

where E is the abstract (conceptual) representation of the set of properties of the systems in D; the ε are injective maps that simulates the ‘mental’ separation of properties from the systems. In the case of coloured systems for example, the condition of commutativity means that if a system appears as red then it possesses the property redness. It is assumed that each element of P_1 which is a predicate in the object language and a name in the property language represents a unique potential property of an individual.

The property space E is a construction characterised by the diagram (4). The E chosen is a natural extension of the set of properties that can be associated to the systems of the domain as reflected in the set of predicates available in the standards of the operational definitions.

The maps $\rho{:}E \to P_1;\ e \mapsto \rho(e)$ can be considered as naming maps for the properties, e.g. a point in abstract space is named by a set of coordinates. To describe the properties we need a formal language, the property language $L(E,P_1 \cup W,R)$, were P_1 denotes the set of names, W the set of variables and R the set of predicates. The property language is associated with the diagrams

$$\begin{array}{ccc} P_1 & \xrightarrow{\gamma} & R \\ \rho\uparrow & \nearrow\chi & \\ E & & \end{array} \qquad (5)$$

where the map ρ symbolises a kind of measurements.

2.3 Theory

A theory for a given domain is the juxtaposition of an object language and a property language. Because of their association the triples of observables δ, π and ρ constitute the bridges between the object language and the property language with the observables δ as the central parts. The diagrams

$$\begin{array}{ccccc} N & \xrightarrow{\pi} & P_1 & \xrightarrow{\phi} & Q \\ \nu\uparrow & \nearrow & \rho\uparrow & \nearrow\chi & \\ D & \xrightarrow[\varepsilon]{} & E & & \end{array} \qquad (6)$$

i.e. the composition of the diagrams (1), (2) and (3), expresses the structure of a scientific theory.

The commutativity of the diagrams (1) and (2) defines a unique π and ρ for each δ and ε. π, ρ and δ all simulates the acts of measurements and will therefore be referred to as observables. Though their function differs the observables in a triple are therefore also given the same name. Colour is an example. Thus, while δ, by $\delta(d) = \text{red}$ associates the colour red to a system d, $\pi(n) = \text{red}$ stands for the atomic proposition “n is red”, $\varepsilon(d) = \text{redness}$ claims that the system possesses the property redness and $\rho(\text{redness}) = \text{red}$ gives the name to the property. The observation that a system is red expressed by the sentence “n is red” is therefore to be interpreted as expressing that the system whose name is n possesses the *property* redness. This interpretation is justified by the commutativity of the diagram (6). The diagram thus shows how the semantic of the property language is based on the operational definitions.

2.3 The Metalanguage of the Object Language

The description of the first order language in the preceding paragraph is done in informal metalanguage. In the following I will proceed to describe the formalisation of the metalanguage in an informal meta-metalanguage.

The metalanguage is denoted $L_G(M_1 \cup M^{(2)},Q)$ where the domain G consists of the set $D \cup L_D(N \cup N^{(2)} \cup V, P_1 \cup P^{(2)} \cup P_2)$ endowed with the directed graph structure defined by (3), $M_1 = D \cup L_D(N \cup N^{(2)} \cup V, P_1 \cup P^{(2)} \cup P_2)$ the names[3] of the

[3] I apply the convention that the symbol(s) representing a term, a sentence, a formula, a node or a relation serves as its name. These objects are only spoken about in the metalanguage not used; they thus do not convey meaning but retain their syntactic

nodes, $M^{(2)}$ the names of the relations q (arrows $d \mapsto n$ etc. in (3)) and Q the predicates of the metalanguage. In the metalanguage D represents the symbolic model of the domain.

The names of the individuals, relations between individuals, terms, sentences and relations between these objects are given by the map

$$\eta : G \to M_1 \cup M^{(2)};$$
$$d \mapsto \eta(d) = d$$
$$n \mapsto \eta(n) = n$$
•
•

$$\left(\nu(d) = n\right) \mapsto \eta\left(\nu(d) = n\right) = (d,n)$$
$$\left(\pi(n) = p\right) \mapsto \eta\left(\pi(n) = p\right) = (n,p) \qquad (7)$$
$$\left(\pi^{(2)}(n_s,n_t) = p^{(2)}\right) \mapsto$$
$$\eta\left(\pi^{(2)}(n_s,n_t) = p^{(2)}\right) = \left((n_s,n_t),p^{(2)}\right)$$
$$\left(\delta(d) = p\right) \mapsto \eta\left(\delta(d) = p\right) = (d,p)$$
$$\left(\delta^{(2)}(r) = p^{(2)}\right) \mapsto \eta\left(\delta^{(2)}(r) = p^{(2)}\right)$$
$$= \left(r,p^{(2)}\right)$$

where $\nu(d) = n$ denotes relations (arrows: $d \mapsto n$) etc.

Each observable α determines an atomic fact about an element of the domain G,

$$\alpha{:}G \to Q;\ g \mapsto \alpha(g) \qquad (8)$$

Moreover, for each observable α there exists a unique map β defined by the condition of commutativity of the diagram

$$\begin{array}{ccc} & \beta & \\ M_1 \cup M^{(2)} & \to & Q \\ \eta \uparrow & \nearrow \alpha & \\ G & & \end{array} \qquad (9)$$

An observable σ, the semantic observable, has the values[4] D, $D^{(2)}$, N, $N^{(2)}$, V, P, $P^{(2)}$, S, H, P_ν, P_π, $P_{\pi^{(2)}}$, P_δ, $P_{\delta^{(2)}}$

$$\sigma{:}G \to Q;$$
$$d \mapsto \sigma(d) = D$$
$$r \mapsto \sigma(r) = D^{(2)}$$
$$n \mapsto \sigma(n) = N$$
$$p \mapsto \sigma(p) = P$$
•
•

$$\left(\nu(d)=n\right) \mapsto \sigma\left(\nu(d)=n\right) = P_\nu \qquad (10)$$
$$\left(\pi(n)=p\right) \mapsto \sigma\left(\pi(n)=p\right) = P_\pi$$
$$\left(\pi^{(2)}(n_s,n_t)=p^{(2)}\right) \mapsto \sigma\left(\pi^{(2)}(n_s,n_t)=p^{(2)}\right)$$
$$= P_{\pi^{(2)}}$$
$$\left(\delta(d)=p\right) \mapsto \sigma\left(\delta(d)=p\right) = P_\delta$$
$$\left(\delta^{(2)}(r)=p^{(2)}\right) \mapsto \sigma\left(\delta^{(2)}(r)=p^{(2)}\right) = P_{\delta^{(2)}}$$

informally defined by[5]

1. Dm, m is an individual
2. $D^{(2)}m$, m is a relation
3. Nm, m is the name of an individual
4. $N^{(2)}m$, m is the name of a relation
5. Vm, m is a variable
6. Pm, m is a 1-ary predicate
7. $P^{(2)}m$, m is a 2-ary predicate
8. Sm, m is a sentence
9. Hm, m is a formula
10. $P_\nu m_1 m_2$, m_1 is named m_2
11. $P_\pi m_1 m_2$, $m_2 m_1$ is a sentence
12. $P_{\pi^{(2)}} m_1 m_2$, $m_2 m_1$ is a sentence
13. $P_\delta m_1 m_2$, m_1 possesses the property referred to by m_2
14. $P_{\delta^{(2)}} m_1 m_2$, m_1 is the relation referred to by m_2

The operational definition is given by the syntactic rules, and interpretation of the language and the semantic value of a symbol are determined by inspection. It should be noticed that these predicates can serve to characterise names and terms of the object language and thus makes possible a map that to a sentence associates a syntactic description of the sentence. The metalanguage might thus serve as

form. Accordingly, self reference and paradoxical sentences are avoided even without the use of distinctive notation.

[4] Notice the reuse of symbols and also that there is a predicate P_δ for each δ etc.

[5] We may refine the notion of sentence by distinguishing between mutually exclusive kinds of sentences.

the basis for the construction of an ontology language.

Strictly speaking, syntactic rules and rules of deduction are formulated in a metalanguage. In the intensional metalanguage the syntactic rules are of the form

atomic sentence: $Nn \wedge Pp \Rightarrow Spn$

conjunction: $Hf_1 \wedge Hf_2 \Rightarrow H(f_1 \wedge f_2)$

univer. quant.: $Hf(x) \Rightarrow H(\forall_x f(x))$

etc.

The rules of deduction, substitution, modus ponens and generalisation are in the notation introduced expressed by [8]

modus ponens: $(Tf_1 \wedge T(f_1 \Rightarrow f_2)) \Rightarrow Tf_2$

generalisation: if it is assumed that the hypotheses underlying the derivation of $f(x)$ does not depend on x then

$$(Hf(x)) \Rightarrow T\forall_x f(x)$$

It is however only the modus ponens that needs be used in the modelling of context aware systems.

4 ONTOLOGIES

Each of the languages is endowed with an ontology that provides implicit definitions of the terms of the vocabularies and at the same time pictures structural properties of the respective domains. The ontology of the object language also provides the background for the representation of the context. Without any specification of the domains, it is only the ontology of the metalanguage that can be given. It is defined by axioms which summarise the content of the commutativity conditions (3):

Axiom: the commutativity conditions (3) hold for an atomic sentence iff the sentence is true, i.e.

$$\begin{pmatrix} Dm_1 \wedge Nm_2 \wedge P_1m_3 \wedge \\ (P_\nu m_1m_2 \wedge P_\delta m_1m_3 \Rightarrow P_\pi m_2m_3) \end{pmatrix} \quad (12)$$
$$\Leftrightarrow m_3m_2 \text{ is true}$$

and similarly for the relations.

Whether an atomic sentence is true or false can be ascertained by inspection using these axioms.

This gives rise to another observable τ given by the values true T, neutral I or false F. τ is neutral for all individuals, relations, terms and formulae, and true or false on the sentences, i.e. if s is a sentence, then the truth of s is expressed by Ts.

5 MODELLING

A context-aware system consists of several elementary systems each of which monitor its environment by means of sensors thus determining its relative state (context) at each moment of time according to the aims the system is designed to satisfy[6]. The elementary systems adapt to the actual state of their environment by means of actuators acting on controllers. Each sensor is thus an observable represented by a map from the domain to the set of possible values of the sensor that to an elementary system associates a value representing a property possessed by the systems and boundaries constituting the environment (relative position, relative velocity, temperature, pressure, …). Similarly, an actuator is an observable represented by a map from the set of elementary systems to the set of values representing the positions of the controller that is acted on by the actuator. The values of the sensors and the actuators are predicates in the object languages for the domain constituted by the total system. Together with the names of the elementary systems they constitute the basic vocabulary for the object language. The ontology of the object language provides the definitions interpreting the empirical data, i.e. the sensor data and the values of the actuators determining the positions of the controllers which together describe the states of the total system. It also contains the knowledge of what is the effect of the positions of the controllers for each elementary subsystem. The possible functional relations between the observables and thus the sensor data are expressed in the ontology of the property language.

The behaviour of the system is determined by externally imposed constraints implicitly represented by control conditions formulated as rules in the metalanguage. The control is based on the observation of truth inherent in the axioms of the metalanguage. If it is no longer true that the state of the system satisfy the control conditions, rules tells which state it should go to. The actual and wanted states are entered into algorithms expressed in the property language. The result of the computation determines actions by the actuators via a set of action rules also formulated in the metalanguage. The intelligence determining the behaviour might be centralised or partly distributed depending on the nature of the system to be constructed.

A car with a cruise is a simple but illustrating example of a context aware system that can be described in accordance with the above

[6] There might also be a central supervising unity communicating with all the elementary systems and keeping track of the states of the system..

modelling scheme. The environment considered is a straight road with slopes. Assume, moreover, that the car possesses two sensors, one measuring the speed relative to the road and the other measuring the gradient of the road beneath the car, and an actuators acting on the accelerator. The control condition states that when the cruise control is set at the velocity v this means that the velocity of the car, as measured by the speedometer, should be in the interval $(v - \Delta v, v + \Delta v)$ for some fixed Δv. The speed of the car moving along the road will start to change whenever the gradient of the slope changes. As the velocity gets smaller than $v - \Delta v$ or bigger than $v + \Delta v$ the velocity control condition is falsified and the cruise control computes the position to be chosen for the accelerator to increase or lower the speed by Δv as a function of the actual slope of the road. The result of the computation is transferred to the action rules that order the actuator to select the given position of the accelerator.

6 FINAL REMARKS

The task of modelling is to represent the structure of a system by means of a symbolism that clearly depicts the essential elements by means of their properties and relations. This, I hope to have shown, can be achieved with respect to the modelling of context aware systems in the given logical framework.

Models might serve as descriptions of existing systems but also as specifications of systems to be constructed. Thus, a model specified in the given logical framework can, except for the algorithms, relatively directly be implemented in OWL/SWRL as part of the construction of a context aware system. In fact, the ontology language based on the intensional metalanguage can be considered as a slight extension of OWL/SWIRL endowed with an alternative semantics [4]. The similarity of the two languages is, moreover, enhanced by the fact that the object language as well as the metalanguage also possesses canonical extensional interpretations obtained by taking the inverse images of the values of the observables as their extensions.

It is also possible to model directly in OWL/SWRL. However, apart from not being able to represent the algorithms which in any case must be implemented by additional means, this language lacks symbolism for the explicit representation of the sensors and activators. Modelling in OWL/SWRL is thus less transparent and controllable and therefore puts stronger demands on the modeller.

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