COLOR IMAGE REGISTRATION AND TEMPLATE MATCHING USING QUATERNION PHASE CORRELATION

B.D.Venkataratana Reddy  
Department of Electronics and Communication Engineering, Madanapalle Institute of Technology & Science,  
Madanapalle-517325, Andhra Pradesh, India.  
E-mail: balam.diguva@gmail.com

Dr. T. Jayachandra Prasad  
Department of Electronics and Communication Engineering, RGM College of Engineering & Technology,  
Nandyal-518501, Andhra Pradesh, India.  
E-mail: jp.talari@gmail.com

ABSTRACT

Due to mathematical limitations, conventional phase correlation technique can only be applied to grayscale images or at most complex images. A full color image must be first converted to a grayscale image before performing the phase correlation, during which the luminance information has been wasted. In this paper, an extension of the phase correlation technique to the quaternion field, based on quaternion Fourier transforms, is proposed. This new technique called quaternion phase correlation (QPC) can make full use of the luminance as well as the chrominance information in color images. The effectiveness of the proposed quaternion phase correlation for color images is demonstrated through its applications in color template matching and color image registration.

Keywords: Colour template matching, Colour image registration, Quaternion Fourier transform.

1 INTRODUCTION

Phase correlation has become a fundamental tool and powerful technique in image processing applications, such as image registration, motion estimation and object recognition. The phase correlation is established on the basis of the Fourier Shift Theorem, aiming to estimate the translational shift between two similar images or sub-images.

Mathematically, phase correlation is defined as

\[
\rho_c = \mathcal{F}^{-1}\left\{ \frac{F \cdot G^*}{|F \cdot G|} \right\}
\]

(1)

where \( F \) and \( G \) are the Fourier transforms of the two images \( f \) and \( g \), respectively. The * in Eq. (1) denotes the complex conjugation, and \( \mathcal{F}^{-1} \) denotes the inverse Fourier transform. The spatial shift between two similar images can be obtained by computing the normalized cross-power spectrum of the two images. The inverse Fourier transform of the normalized cross-power spectrum yields a complex image, the modulus of which defines a 2-D surface with a delta function (also referred to as the peak) at the position corresponding to the spatial shift between the two images.

However, the algebra of the phase correlation only works with real numbers or at most complex numbers, thus limiting its application in color image processing. To apply the phase correlation to color images, it is necessary to convert them to grayscale images, which will lead to the waste of the chrominance information, and consequently the failure in some applications. For example, in the color template matching application, a candidate of the same shape but of different colors might be considered to be the true match, because the chrominance information has been ignored. Therefore, an extension of the phase correlation is required to make full use of the luminance and the chrominance information of color images.

Quaternions, as an extension of the complex algebra, may help meet this demand. A quaternion number has four components, a real part and three imaginary parts, which naturally coincides with the three components of a color pixel and can address our problems. In this paper, we propose an extension of the conventional phase correlation technique to the quaternion field.

2 QUATERNIONS

The concept of the quaternion was introduced by Hamilton in 1843 [1]. It is the generalization of a
complex number. A complex number has two components: the real and the imaginary part. The quaternion, however, has four components, i.e., one real part and three imaginary parts and can be represented in Cartesian form as:

\[ q = w + xi + yj + zk \]  

(2)

where \( w, x, y \) and \( z \) are real numbers and \( i, j \) and \( k \) are complex operators which obey the following rules.

\[
\begin{align*}
ij &= k, \quad jk = i, \quad ki = j, \\
ji &= -k, \quad kj = -i, \quad ik = -j
\end{align*}
\]

and also satisfies \( i^2 = j^2 = k^2 = ijk = -1 \). From these rules, it is clear that multiplication is not commutative.

The quaternion conjugate is \( q = w - xi - yj - zk \) and the modulus of a quaternion is given by

\[ |q| = \sqrt{w^2 + x^2 + y^2 + z^2} \]  

(3)

A quaternion with zero real part is called a pure quaternion and a quaternion with unit modulus is called a unit quaternion. The imaginary part of a quaternion has three components and may be associated with a 3-space vector. For this reason, it is sometimes useful to consider the quaternion as composed of a vector part and a scalar part. Thus \( q \) can be expressed as

\[ q = S(q) + V(q), \]  

(4)

where the scalar part, \( S(q) \), is the real part i.e. \( S(q) = w \) and the vector part is a composite of three imaginary components,

\[ V(q) = xi + yj + zk. \]

The product of two quaternions expressed in terms of their scalar and vector parts is given by

\[ qp = S(q)S(p) - V(q)V(p) + S(q)V(p) + S(p)V(q) + V(q) \times V(p) \]  

(5)

where , and \( \times \) denotes the vector dot cross products, respectively. It follows from this that the dot and cross products of two pure quaternions \( m \) and \( n \) are given by

\[
\begin{align*}
m \cdot n &= -\frac{1}{2} (mn + nm) \\
m \times n &= \frac{1}{2} (mn - nm)
\end{align*}
\]

(6)

3 QUATERNION REPRESENTATION OF COLOR IMAGE PIXELS

Color image pixels have three components, and they can be represented in quaternion form using pure quaternions [2]. For images in RGB colour space, the three imaginary parts of a pure quaternion can be used to represent the red, green and blue colour components. For example, a pixel at image coordinates \((x, y)\) in an RGB image can be represented as

\[ f(x, y) = r(x, y)i + g(x, y)j + b(x, y)k \]  

(7)

where \( r(x, y) \), \( g(x, y) \) and \( b(x, y) \) are the red, green and blue components of the pixel, respectively.

Using quaternions to represent the RGB color space, the three color channels are processed equally in operations such as multiplication. The advantage of using quaternion based operations to manipulate color information in an image is that we do not have to process each color channel independently, but rather, treat each color triple as a whole unit. We believe, by using quaternion operations, higher color information accuracy can be achieved because a color is treated as an entity. In quaternion multiplication, each of the three imaginary components is multiplied in a similar manner with other components.

4 DISCRETE QUATERNION FOURIER TRANSFORM

Based on the concept of quaternion multiplication and exponential, the Quaternion Fourier Transform (QFT) has been introduced. Due to the non commutative multiplication rule of quaternion algebra, there are several forms of quaternion Fourier transforms. We adopted the form presented in the work of [2] which divides the discrete QFT into two categories, namely the right-side form and the left-side form.

Discrete version of the right-side and left-side quaternion Fourier transforms can be represented as

\[ F^R = F^R(u, v) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-\mu 2\pi (\frac{xy}{MX} + \frac{yn}{NY})} \]  

(8)

\[ F^L = F^L(u, v) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} e^{-\mu 2\pi (\frac{xy}{MX} + \frac{yn}{NY})} f(x, y) \]  

(9)

Similarly, the inverse quaternion Fourier transforms can be denoted as:

\[ F^{-R} = f(x, y) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} F(u, v) e^{\mu 2\pi (\frac{xy}{MX} + \frac{yn}{NY})} \]  

(10)
\[ F^{-1} = f(x, y) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} e^{i2\pi \left( \frac{xy}{M} + \frac{yv}{N} \right)} f(u, v) \]  

(11)

In this transform, the hypercomplex operator was generalized; \( \mu \) is any unit pure quaternion. \( \mu \) determines a direction in color space and an obvious choice for color images is the direction corresponding to the luminance axis which connects all the points \( r=g=b \).

In RGB color space this is the “gray line” and \( \mu \) would be \( (i+j+k)/\sqrt{3} \). In this paper the transform in Eq. (8) is denoted by \( F^R \), its reverse in Eq. (10) by \( F^{-R} \) and the related transpose transform, with hypercomplex exponential on the left, by \( F^L \) and its reverse by \( F^{-L} \). Computing the cross power spectrum of two hypercomplex Fourier transformed images requires the decomposition of a quaternion into its parallel and perpendicular components with respect to a pure quaternion (axis).

5 QUATERNION PHASE CORRELATION

5.1 Definition of quaternion cross – correlation

The cross-correlation of two images \( f(x, y) \) and \( g(x, y) \) was originally extended to quaternion images using basic quaternion arithmetic.

\[ c(m, n) = \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} f(p, q) g(p-m, q-n) \]  

(12)

where the shift operation on \( g(x, y) \) is implemented cyclically using modulo arithmetic [3]. \( c(m, n) \) is the correlation function of the images. If the images \( f(x, y) \) and \( g(x, y) \) are the same images the autocorrelation of the images is computed. If the mean, or DC level of each image is subtracted first, the cross-covariance is obtained. Direct evaluation of the cross correlation function is impractical for all but the smallest images due to the high computational cost.

5.2 Definition of quaternion phase correlation

The hypercomplex or quaternion generalization of phase correlation, for use with colour images was first presented in [3]. It is reasonable to seek for simple relations between the QFT and the quaternion phase correlation, so that we can use efficient QFT algorithms to implement correlation in frequency domain rather in spatial domain.

It is possible to define quaternion phase correlation by a two-stage process, by first computing the cross-correlation using the Eq. (12), and then using Quaternion Fourier Transform equation to compute a cross- power spectrum, which may then be normalized. The inverse transform of this normalized spectrum then yields a quaternion phase correlation. However, we now present a more direct method directly analogous to general mathematical phase correlation equation by presenting a new formula for cross-correlation, in which the quaternion or hypercomplex cross-power spectrum is computed as an intermediate result [4]:

\[ R_R(m, n) = F_L(u, v) G_R^R(u, v) + F_{-L}(u, v) G_{-R^R}(u, v) \]  

(13)

where \( F_L(u, v) = F^L[f(x, y)] \), and \( F_{-L}(u, v) = F^{-L}[f(x, y)] \) and similarly for \( G \). This depends on the decomposition of \( G_R(u, v) \) into components parallel and perpendicular to \( \mu \).

Apart from this being defined as a quaternion operation, it is otherwise a straight forward resolution of a vector into a direction parallel to \( \mu \) and a plane normal to \( \mu \). Given this definition of the cross-power spectrum, we can obtain the cross correlation as

\[ r(m, n) = F^{-R}[R_R(m, n)] \]  

(14)

and the quaternion or hypercomplex phase correlation as

\[ p(m, n) = F^{-R} \left( \frac{R_R(m, n)}{|R_R(m, n)|} \right) \]  

(15)

without requiring the cross-correlation to be computed.

5.3 Algorithm to implement quaternion phase correlation

The algorithm to implement quaternion phase correlation can be broken down into the following steps.

1. Given two input images \( f(x, y) \) and \( g(x, y) \). Compute the Quaternion Fourier Transform (QFT) of both the images as:

\[ F_L(u, v) = F^L[f(x, y)] \quad \text{and} \quad G_R(u, v) = G^R[g(x, y)] \]

2. Compute the inverse QFT of the image \( f(x, y) \) as:

\[ F_{-L}(u, v) = F^{-L}[f(x, y)] \]

3. Decompose \( G_R(u, v) \) into components parallel and perpendicular to the transform axis \( \mu \).

\[ G_{R\parallel}(u, v) = \frac{1}{2} \left( G_R(u, v) - \mu \cdot G_R(u, v) \mu \right) \]

\[ G_{R\perp}(u, v) = \frac{1}{2} \left( G_R(u, v) + \mu \cdot G_R(u, v) \mu \right) \]
4. Compute the conjugates of \( F_{g}(u,v) \) and \( F_{-L}(u,v) \) to obtain \( \overline{F_{g}(u,v)} \) and \( \overline{F_{-L}(u,v)} \).

5. Compute the hypercomplex cross-power spectrum of the two images as

\[
R_{g}(m,n) = \overline{F_{g}(u,v)}G_{R_{L}}(u,v) + \overline{F_{-L}(u,v)}G_{R_{L}}(u,v)
\]

6. Normalize the cross-power spectrum by dividing it with its modulus element wise as

\[
\frac{R_{g}(m,n)}{|R_{g}(m,n)|}
\]

7. Obtain the hypercomplex phase correlation by applying inverse QFT to the normalized cross-power spectrum.

\[
p(m,n) = F^{-r}\left\{ \frac{R_{g}(m,n)}{|R_{g}(m,n)|} \right\}
\]

6 APPLICATIONS AND EXPERIMENTS

6.1 Color template matching

In colour template matching application, the proposed quaternion phase correlation can be used as a measurement of structural and colour similarity between the template image and the candidate image. The peak location of the resulting phase correlation surface corresponds to the matched image area. Given a template colour image \( g(x, y) \) and input colour image \( f(x, y) \), the quaternion phase correlation can be calculated using Eq. (15). The difference between the quaternion phase correlation based and conventional phase correlation-based template matching lies in that conventional phase correlation based matching only considers candidate areas with the largest similarity, while the quaternion phase correlation based matching also takes the colour similarity into consideration.

The synthetic images shown in Fig.1 and Fig.2 are used to illustrate our method. The artificial car shown in Fig.1(b) is the colour template. Fig.1(a) is the input colour image to be searched for the true match area. There are five candidate cars in true input image which are of the same shape and size but of different colours. Only the candidate car marked 1 is the true match of the template. Following our quaternion phase correlation based matching algorithm, the phase correlation surface is obtained as shown in Fig.1(c). From this figure, it can be observed that only one outstanding peak exists, which corresponds to the true match area.

To further analyze the performance of quaternion phase correlation based matching technique, we also give the result of the conventional phase correlation based matching scheme for comparison. The matching is done by first converting the input colour image and template image into grayscale images, and then computing the conventional phase correlation. Fig.1(f) illustrates the phase correlation surface generated by the conventional phase correlation. From this figure, it can be observed that there are five peaks with approximately the same height which makes it difficult to identify the match from others.

The result is not unexpected. Due to the mathematical limitation of the standard phase correlation, chrominance information is wasted during the conversions of input colour images and templates into grayscale images, which causes the failure of matching in this case. While the quaternion phase correlation-based matching takes advantages of the quaternion algebra and makes full use of the input information, thus can gain better matching performance.

Fig.2 shows the colour template matching in a database image containing different letters of various colours. The template is a dark green coloured capital letter A. The task is to detect the location of the template in the database image. Fig.2(c) shows the phase correlation surface using quaternion phase correlation based method. From the figure, it can be observed that the green letter A corresponds to the highest peak because it is most similar to the template, based on the structural and colour information in combination. This quaternion phase correlation algorithm successfully identifies the candidate location of the template by estimating the pattern and colour information as a whole unit. Fig.2(f) illustrates the phase correlation surface generated by the conventional phase correlation. From this figure, it can be observed that there are peaks with approximately the same height which makes it difficult to identify the match from others.

6.2 Color image registration

Image registration is the process of overlaying images (two or more) of the same scene taken at different times, from different viewpoints, and/or by different sensors. The registrations geometrically align two images (the reference and sensed images).

Image registration has been gaining research interests over the past 10 years, while algorithms that can deal with colour images directly are seldom proposed. In this paper, with the proposed quaternion phase correlation, we are able to extend some conventional phase correlation based registration algorithms for direct colour image registration.

The key issue in image registration is to estimate the spatial shift between the two images to be registered. We can use the expression given in Eq. (15) to implement the estimation.
Figs. 3(b) & (c) show two overlapping subimages of a flower image. There are common areas in the two images and there are also areas not in common. Both the sub-images are generated with a spatial shift of (10, 10) between them. Fig. 3(b) has an added Gaussian noise of zero mean and variance 0.125 and Fig. 3(c) has an added Gaussian noise of zero mean and variance 0.225. Fig. 3(d) is a plot of the modulus of the hypercomplex phase correlation surface. It clearly shows an impulse and this impulse occurs at a position corresponding exactly to the relative shift in pixels between Figs. 3(b) and (c). The peak has amplitude of 0.0658 and the mean of the phase correlation surface is 0.0031. Quantitative experiments have been performed on the two subimages for various spatial shifts between them using the registration scheme based on the proposed quaternion phase correlation algorithm and also conventional phase correlation based method. In the later method the registration is performed by first converting the input colour images into grayscale images, and then computing the conventional phase correlation. Fig. 4(c) illustrates the phase correlation surface obtained using conventional PC. From this figure it can be seen that the location of the peak is difficult to identify, though it occurs at a position corresponding to the relative shift in pixels between Figs. 4(a) and (b). The results of both methods are listed in Table 1. From this table it can be seen that the performance of the QPC algorithm is better compared to the conventional PC. The amplitudes of peaks in the phase correlation surface obtained using QPC algorithm are larger than those obtained with conventional PC.

7 CONCLUSION

In this paper, an extension of conventional phase correlation to the quaternion field has been presented. Using the proposed QPC, we can gain the advantage of processing a colour image in a holistic manner without wasting chrominance information as following the conventional PC based methods.

Applications in colour template matching and colour image registration have shown its great potentials. And we would like to point out that, the proposed QPC is not limited to this, but also can be applied to other colour image processing fields, such as object recognition and target tracking.

(a)                                 (b)                                                           (c)

(d)                                                (e)                                                             (f)

Figure 1: Colour template matching in synthetic images (a) the input colour image (b) the colour template (c) phase correlation surface using quaternion phase correlation (d) and (e) the input images with grayscale information (f) phase correlation surface using the conventional phase correlation.
Figure 2: Colour template matching in the database image containing a number of different letters and colours. (a) the input colour image (b) the target green letter A (c) phase correlation surface using quaternion phase correlation (d) and (e) the input images with grayscale information (f) phase correlation surface using the conventional phase correlation.

Table 1: Performance comparison of QPC and Conventional PC.

<table>
<thead>
<tr>
<th>Spatial shift</th>
<th>Quaternion phase correlation surface peak amplitude</th>
<th>Estimated spatial shift</th>
<th>Conventional phase correlation surface peak amplitude</th>
<th>Estimated spatial shift</th>
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<td>0.0183</td>
<td>(10,10)</td>
</tr>
</tbody>
</table>
Figure 3: Image registration using Quaternion phase correlation (a) original colour image (b) and (c) subimages generated with a spatial shift of $(10,10)$ (d) phase correlation surface.

Figure 4: Image registration using conventional phase correlation (a) and (b) subimages with grayscale information generated with a spatial shift of $(10,10)$ (c) phase correlation surface.

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