Modeling the Effect of Clipping and Power Amplifier Non-Linearities on OFDM Systems

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ABSTRACT

The high Peak to Average power Ration (PAR) levels of Orthogonal Frequency Domain Multiplexing (OFDM) signals forces to the utilization of linear power amplifiers. However, linear amplifiers have low power efficiency which is problematic considering that battery life is a critical resource in mobile systems; also linear amplifiers have a clipping effect when considered as a limiter. Amplifier nonlinearities affect drastically the performance of OFDM systems. In this paper, the effects of Clipping and amplifier nonlinearities are modeled in an OFDM system. We showed that the distortion due to these effects is highly related to the dynamic range it self rather than the clipping level or the saturation level of the nonlinear amplifier. Computer simulations of the OFDM system using Matlab are completely matched with the deduced model in terms of OFDM signal quality metrics such as BER, ACPR and EVM.

1. INTRODUCTION

In OFDM systems, the combination of different signals with different phase and frequency give a large dynamic range that is used to be characterized by a high PAR, which results in severe clipping effects and nonlinear distortion if the composite time signal is amplified by a power amplifier, which have nonlinear transfer function. This degrades the performance of an OFDM system. A measure of the degradation can be very helpful in evaluating the performance of a given system, and in designing a signaling set that avoids degradation. The high PAR sets strict requirements for the linearity of the PA. In order to limit the adjacent channel leakage, it is desirable for the PA to operate in its linear region. High linearity requirement for the PA leads to low power efficiency and therefore to high power consumption. PAs are divided into classes according to the biasing used. A class A amplifier is defined as an amplifier that is biased so that the current drawn from the battery is equal to the maximum output current. The class A amplifier is the most linear of all amplifier types, but the maximum efficiency of the amplifier is limited to 50%. In reality, due to the fact that the amplitude of the input signal is most of the time much less than its maximum value, the efficiency is much less than the theoretical maximum, i.e. only a few percent. This poor efficiency causes high power consumption, which leads to warming in physical devices. This is a problem especially in a base station where the transmitted power is usually high. To achieve a better efficiency, the amplifier can be biased so that current flows only half the time on either the positive or negative half cycle of the input signal. An amplifier biased like this is called a class B amplifier. The cost of the increased efficiency is worse linearity than in a class A amplifier. High demands on linearity make class B unsuitable for a system with high PAR. On the other hand, the large scale of the input signal makes it difficult to bias an amplifier operating in class A. In practice, the amplifier is a compromise between classes A and B, and is called a class AB amplifier. [1]

Several options appear in the literature related with OFDM systems and nonlinearities. PAR reduction using clipping or coding or phase optimization techniques or a combination of any two of them [2], are the tools to combat nonlinearities used in the transmitter. Also a good work have been done in modeling the performance of OFDM systems with power amplifiers in [1], but it have assumed the OFDM signal to have Gaussian distribution which is not very accurate description of the composite time OFDM signal. In this paper, the effect of power amplifier nonlinearities is modeled in OFDM systems. Section 2 depicts OFDM signal statistical properties while Section 3 introduces power amplifier models. The derivation of clipping noise and distortion is presented in Section 4, OFDM signal quality metrics are discussed in Section 5. Simulation results are included in Section 5. Finally, the conclusions are drawn.

2. OFDM SIGNAL STATISTICAL PROPERTIES

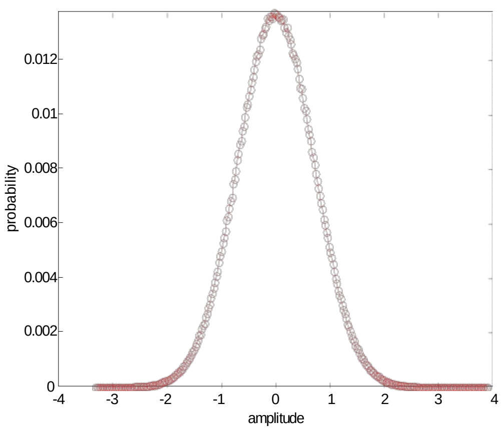
It is well known that according to the central limit theory that, the real and imaginary parts of the OFDM signal completely agree with the normal distribution and consequently its absolute agrees with the Rayleigh distribution with Probability density function expressed by:

$$P(x) = \frac{x}{s} e^{-\frac{x^2}{2s^2}} \quad , x \in [0, \infty]$$

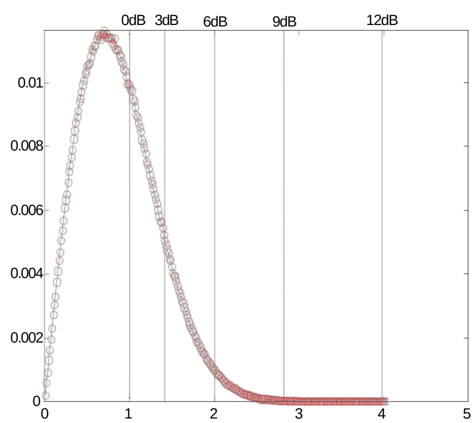
Where s is a parameter, and Mean $\mu = s\sqrt{\pi/2}$ and variance

$$\sigma^2 = \frac{4-\pi}{2} s^2$$

Figure (1) listed below explicitly shows that the measured amplitude histogram of the (a) in-phase component/Quadrature component and (b) amplitude of a 256-subcarrier OFDM signal.



(a) in-phase /Quadrature component histogram



(b) Amplitude histogram

Figure (1) histogram of a 256-subcarrier OFDM signal

It is clear that the distribution in figure (1-a) obeys a Gaussian distribution while that in figure (1-b) Rayleigh distribution.

3. POWER AMPLIFIER MODEL

A short description of power amplifier models will be given in this section. Consider an input signal in polar coordinates as [1]

$$x = \rho e^{j\phi}$$

The output of the power amplifier can be written as

$$g(x) = M(\rho) e^{j(\phi + P(\rho))}$$

Where M(ρ) represents the AM/AM conversion and P(ρ) the AM/PM conversion characteristics of the power amplifier.

Several models have been developed for nonlinear power amplifiers, the most commonly used ones are as follows

3.1. Limiter Transfer characteristics

A Limiter (clipping) amplifier is expressed as [4]:

$$M(\rho) = \begin{cases} \rho & |\rho| < A \\ A & |\rho| \geq A \end{cases} \qquad (1)$$

Where ***A*** is the clipping level. This model does not consider AM/PM conversion.

3.2. Solid-State Power Amplifier (SSPA)

The conversion characteristics of solid-state power amplifier are modeled by Rapp's SSPA with characteristic [4]:

$$v_{out} = \frac{v_{in}}{\left(1 + \left(|v_{in}| / v_{sat}\right)^{2p}\right)^{\frac{1}{2p}}} \qquad (2)$$

Where v_{out} and v_{in} are complex i/p & o/p, v_{sat} is the output at the saturation point ($v_{sat} = A/\sqrt{2}$), and P is "knee factor" that controls the smoothness of the transition from the linear region to the saturation region of characteristic curve (a typical value of *P* is 1).

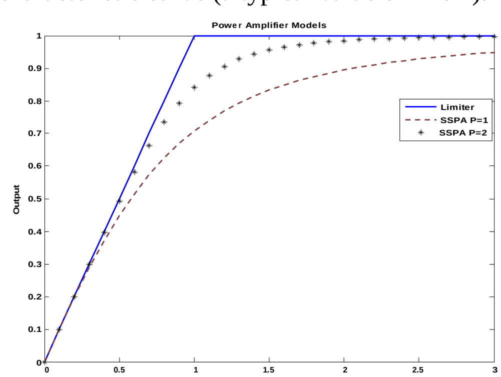


Figure (2) AM/AM conversion of HPA

Figure (2) shows the AM/AM conversion of the two described models with ***A=1.*** It is clear from the figure; as the value of knee factor increases the SSPA model approaches the Limiter Model.

One problem with these methods is that the special model of nonlinear device requires expensive and time consuming experimental measurements to identify model parameters. On the contrary, simple measures of nonlinearity directly related to a low-order polynomial model, such as third and fifth-order intersect points, are usually available to a system designer at the early stage of specification definition or link budget analysis. The third order nonlinearity model can be described by the Taylor series as [3]:

$$y(t) = a_o + a_1 x(t) + a_2 \, x^2(t) + a_3 x^3(t)$$

Where; *x(t)* is the input, *y(t)* is the output signal and $\{a_n\}$are Taylor series coefficients.

The nonlinearity of radio frequency circuits is often expressed in terms of the third-order intercept point (A_{IP3}). It can be shown that A_{IP3} and parameters of third order nonlinearity model are related as [3]:

$$A_{IP3} = \sqrt{\frac{4a_1}{3|a_3|}}$$

In this study, we only consider the third-order model of nonlinearity, since the third-order nonlinearity is usually dominated in real systems.

In this paper we have simulated the Rapp's SSPA model, and then deduced the third order nonlinearity model to formulate the relation between v_{sat} in the Rapp's SSPA model and A_{IP3} in dB widely used to express nonlinearity and we have got:

$$v_{sat} = p_1 (A_{IP3})^3 + p_2 (A_{IP3})^2 + p_3 (A_{IP3}) + p_4$$

Where

$$p_1 = 0.001936, \; p_2 = -0.09044,$$
$$p_3 = 1.438, \; p_4 = -5.197$$

4. NONLINEARITY DISTORTION ANALYSIS

Here we will analyze the effect of nonlinear amplifier on the OFDM signal, first: we will consider the NLA as a limiter that is expressed by:

$$g(x) = \begin{cases} x & x \le a \\ a & x > a \end{cases}$$

And thus the distortion due to the NLA as a limiter can be represented by an extra Gaussian noise with variance $\sigma^2_{\lim iter}$ where

$$\sigma_{\lim iter} = \int_a^\infty (x-a)^2 P(x)dx = \int_a^\infty (x-a)^2 \; \frac{x}{s} e^{-\frac{x^2}{2s^2}} dx$$

$$\therefore \sigma_{\lim iter} = \frac{1}{s}\left[\begin{array}{l} e^{-\frac{x^2}{2s^2}}(-x^2 s^2 - (a^2 + 2s^2)s^2 + 2axs^2) \\ - a\sqrt{2\pi}\, s^3 erf(\frac{x}{2s}) \end{array}\right]_a^\infty$$

$$= \frac{1}{s}\left(0 - a\sqrt{2\pi}\, s^3\right) - \frac{1}{s}\left(\begin{array}{l} e^{-\frac{a^2}{2s^2}}(-a^2 s^2 - (a^2 + 2s^2)s^2 \\ + 2a^2 s^2) - a\sqrt{2\pi}\, s^3 erf(\frac{a}{2s}) \end{array}\right)$$

This leads to

$$\sigma_{\lim iter} = a\sqrt{2\pi}\, s^2 (erf(\frac{a}{\sqrt{2s}}) - 1) + 2s^3 e^{-\frac{a^2}{2s^2}}$$

And finally

$$\sigma_{\lim iter} = 2s^3 e^{-\frac{a^2}{2s^2}} + a\sqrt{2\pi}\, s^2 erfc(\frac{a}{\sqrt{2s}}) \qquad (3)$$

If we consider the solid state power amplifier model given by Rapp:

$$g(x) = \frac{x}{(1 + (\frac{x}{v_{sat}})^{2p})^{\frac{1}{2p}}}$$ And let p=1 and $v_{sat} = a$

The NLA distortion can be represented by

$$\sigma_{SSPA} = \int_0^\infty (x - g(x))^2 P(x)dx$$

$$= \int_0^\infty (x - \frac{x}{\sqrt{1 + (\frac{x}{v_{sat}})^2}})^2 \frac{x}{s} e^{-\frac{x^2}{2s^2}} dx$$

$$\sigma_{SSPA} = \frac{1}{2s}\left(\begin{array}{l} -2a^2 s^2 + 4s^4 + a^4 e^{\frac{a^2}{2s^2}} Ei\left[-\frac{a^2}{2s^2}\right] - \\ 2\sqrt{a^2} e^{-\frac{a^2}{2s^2}} \sqrt{2\pi}\, s(a^2 - s^2) erf\left[\frac{a}{\sqrt{2s}}\right] + \\ 2\sqrt{a^2} e^{-\frac{a^2}{2s^2}} \sqrt{2\pi}\, s(a^2 - s^2) \end{array}\right)$$

Where Ei(x) is the Exponential integral, is defined as

$$E_i(x) = \int_{-\infty}^{x} \frac{e^t}{t} dt$$ It is plotted as in figure (3)

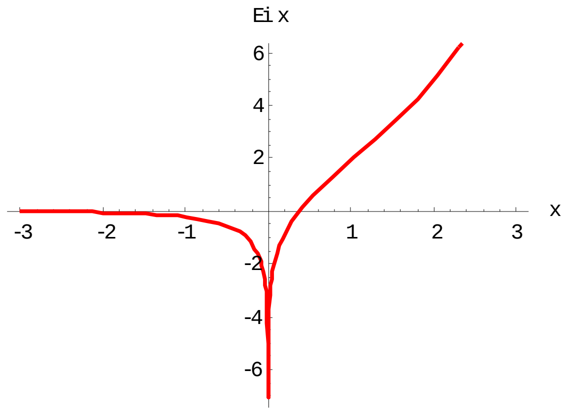


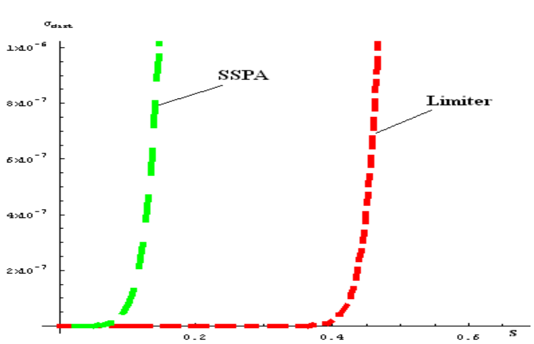
Figure (3) Exponential integral

$$\therefore \sigma_{SSPA} = \frac{1}{2s}\left(\begin{array}{l} 2ae^{-\frac{a^2}{2s^2}}\sqrt{2\pi}\, s(a^2 - s^2)\left(1 - erf\left[\frac{a}{\sqrt{2}s}\right]\right) \\ + a^4 e^{-\frac{a^2}{2s^2}} Ei\left[-\frac{a^2}{2s^2}\right] - 2a^2 s^2 + 4s^4 \end{array} \right)$$

And finally

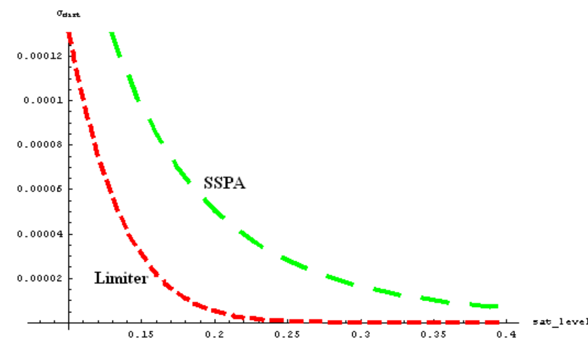
$$\sigma_{SSPA} = \left(\begin{array}{l} e^{-\frac{a^2}{2}}\sqrt{} \qquad \left[\frac{a}{\sqrt{2}s}\right] \\ \frac{a^4}{2s} e^{-\frac{a^2}{2s^2}} Ei\left[-\frac{a^2}{2s^2}\right] - a^2 s + 2s^3 \end{array} \right) \quad (5)$$

When plotting the deduced distortion models in eq. (4, 5) versus the distribution parameter (s) with saturation level a= 2 (A_{IP3} = 10 dB) we notice as shown in figure (4-a) below that the distortion due to the SSPA non-linearity is much more larger than that of its limiting effect, also it is obvious that the distortion is highley senstive to any variation of the parameter (s) as the slopes of the curves show.

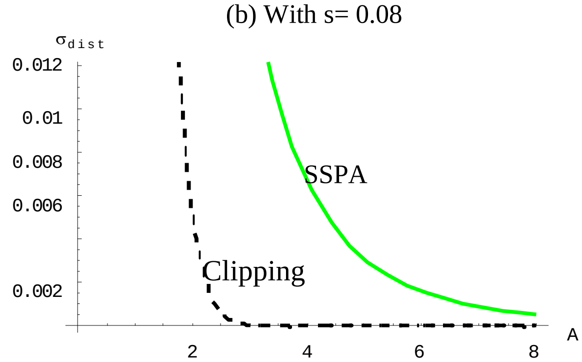


(a) With saturation level a= 2 (A_{IP3} = 10 dB)

On the other hand, when plotting distortion model versus the saturation level with parameter (s=0.08) as depicted in figure (4-b), it is shown that the distortion decays as the value of saturation level increases. And when plotting distortion model versus the saturation level with parameter (s=0.8) figure (4-c) shows a great increase in the distortion despite of the constant value of (A/s).

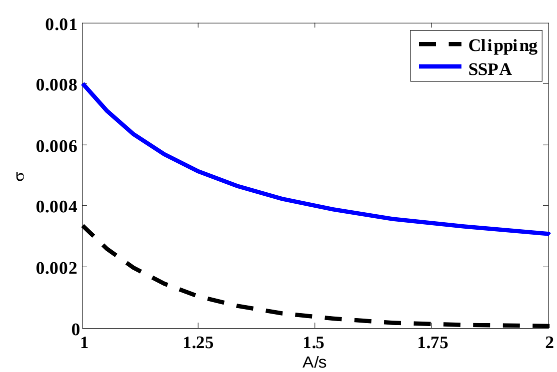


(b) With s= 0.08



(c)With s=0. 8

and finally when plotting distortion model versus (A/s) as shown in figure(4-d) that shows that the distortion is reduced as the value of (A/s) increases.



(d) Distortion Vs. A/s

Figure (4) Non-linear Amplifier Distortion

From (4-a,b,c,and d) it is clear that the distortion due to these effects is highly related to (s) the distribution parameter, that controls the dynamic range it self, rather than the clipping level or the saturation level of the nonlinear amplifier.

5. OFDM THE SIGNAL QUALITY METRICS

5.1. Error Vector Magnitude

The modulation accuracy of the OFDM signal is measured by Error Vector Magnitude. EVM is a measure for the difference between the theoretical wave and modified version of the measured waveform. The measured waveform is modified by first passing it through a specified receiver measuring filter.
The waveform is further modified by selecting the frequency, absolute phase, absolute amplitude and clock timing so as to minimize the error vector. The EVM result is defined as the square root of the ratio of the mean error vector power to the mean reference signal power expressed as a percentage. Mathematically, the error vector e can be written as

e = y-x;

Where y is the modified measured signal and x the ideal transmitted signal. EVM can be defined as

$$EVM_{rms} = \sqrt{\frac{E[|e|^2]}{E[|x|^2]}}$$

5.2. The Adjacent Channel leakage Power Ratio (ACPR)

Another figure of merit, specific to evaluate the out of band behavior of the HPA, is the ACPR; it should stay below the value specified. The ACPR is the ratio of the transmitted power to the power after a receiver filter in the adjacent channel.
In order to evaluate the ability of HPA models to reproduce the ACPR, we will use:

$$\Delta\ ACPR = ACPR(\tilde{S}(f)) - ACPR(S(f))$$

Where $\tilde{S}(f)$ is the PSD of the output of HPA $S(f)$ is the true output.

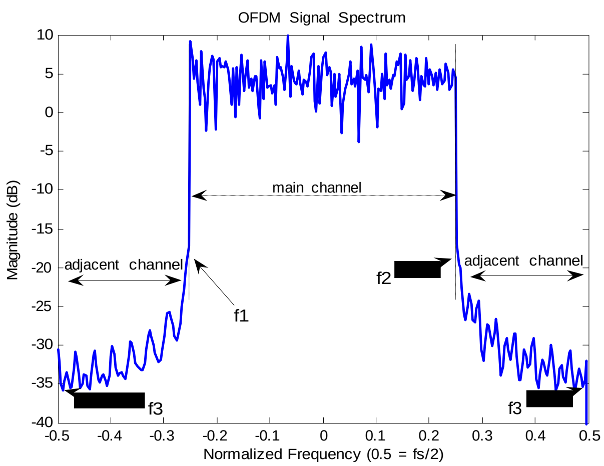


Figure (5) OFDM signal spectrum

The ACPR can be defined as:

$$ACPR = \frac{\int_{f2}^{f3} S(f)df}{\int_{f1}^{f2} S(f)df} + \frac{\int_{f1}^{f4} S(f)df}{\int_{f1}^{f2} S(f)df}$$

Where f_1 and f_2 are the frequency limits of the main channel, and f_2 and f_3 are the frequency limits of the upper adjacent channel, and f_1 and f_4 are the frequency limits of the lower adjacent channel.

6. SIMULATION RESULTS

An OFDM system is implemented using 512 carriers with cyclic prefix length equal to 4. Each carrier is modulated using 16-QAM constellation. AWGN noise is included. BER simulations compared with theoretical results considering the power amplifier distortion models deduced above in equations (4, 5) are shown in Figure 6. From this figure, it is possible to see that it was predicted in the previous analysis. The effect of nonlinear power amplifier is illustrated in figure 6, where a limiter amplifier is included in the simulations with clipping levels of 12 *dB*. The harmful effect of the nonlinearity can also be clearly seemed in this figure. And finally the figure shows that the computer simulations of BER are completely matched with the deduced models both the limiting and the nonlinearity effect.

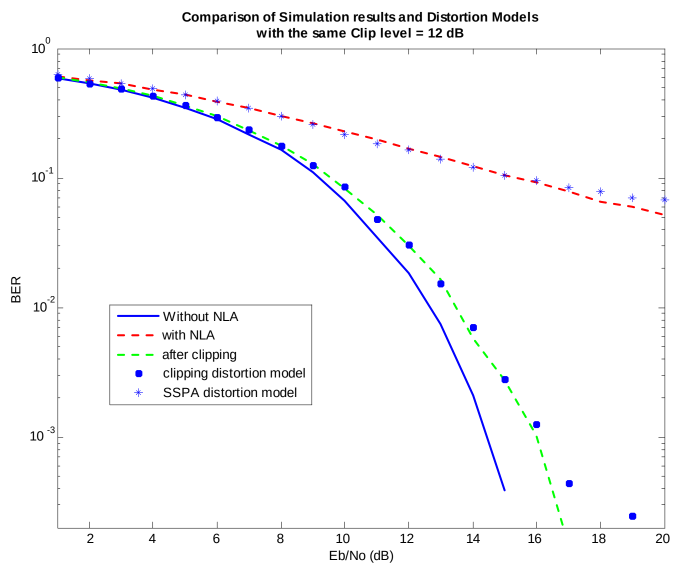


Figure (6) BER of OFDM system with HPA

Table (1) shows The ACPR value for both limiting and non-linear effects with different limiting values relative to the maximum absolute value of the OFDM composite time signal **Ymax**, it is clear that as the limiting value decreases the ACPR increases. It can also be noted that the effect of nonlinearity on ACPR value is negligible as compared to that of limiting as the clip level varies; this is due to the fact that the spectral leakage that causes the ACPR to increase is mainly due to the clipping that can be viewed as windowing the spectrum by rectangular window.

Table (1)
ACPR value with different limiting values

Saturation Level	Limiting (Clipping)	Nonlinearity
Ymax	-44.7094 dB	-28.7268 dB
2 Ymax	-44.7094 dB	-31.0497 dB
10 Ymax	-44.7094 dB	-32.8791 dB
0.75 Ymax	-30.4841 dB	-27.2594 dB
0.5 Ymax	-23.4736 dB	-25.0624 dB
0.25 Ymax	-14.5232 dB	-22.3549 dB

Table (2) shows The EVM value for both limiting and non-linear effects with different limiting values, it is clear that as the limiting value decreases the EVM increases. And as the EVM is a measure of the total distortion it is highly affected by the nonlinearity rather than the limiting effect.

Table (2)
EVM value with different limiting values

Saturation Level	Limiting (Clipping)	Nonlinearity
Ymax	1.3110e-016	0.0929
2 Ymax	1.3110e-016	0.0273
10 Ymax	1.3110e-016	0.0012
0.75 Ymax	0.0126	0.1448
0.5 Ymax	0.0969	0.2474
0.25 Ymax	0.3972	0.4741

7. CONCLUSIONS

In this paper, the effects of nonlinearities in the power amplifier over OFDM systems were analyzed and simulated. We can conclude that:

- The distortion due to high power amplifier, either limiting or nonlinearity effects, is highly related to the distribution parameter (s), that controls the dynamic range it self, rather than the clipping level or the saturation level of the amplifier.
- Also it is noticed that the effect of nonlinearity on ACPR value is negligible as compared to that of limiting as the clip level varies; this is due to the fact that the spectral leakage that causes the ACPR to increase is mainly due to the clipping that can be viewed as windowing the spectrum by rectangular window.
- The EVM is a measure of the total distortion it is highly affected by the nonlinearity rather than the limiting effect. And generally as the limiting value decreases the EVM increases.

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